CPT violation in entangled $B^0 - \overline{B}^0$ states and the demise of flavour tagging

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Abstract

We discuss the demise of flavour tagging due to the loss of the particle-antiparticle identity of neutral B-mesons in the Einstein-Podolsky-Rosen correlated states. Such a situation occurs in cases where the CPT operator is ill-defined, as happens, for example, in quantum gravity models with induced decoherence in the matter sector. The time evolution of the perturbed $B^0 - \overline{B}^0$ initial state, as produced in B-factories, is sufficient to generate new two-body states. For flavour specific decays at equal times, we discuss two definite tests of the two body entanglement: (i) search for the would-be forbidden $B^0 B^0$ and $\overline{B}^0 \overline{B}^0$ states; (ii) deviations from the indistinguishable probability between the permuted states $\overline{B}^0 B^0$ and $B^0 \overline{B}^0$.

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The determination of the initial flavour of a single neutral meson is usually referred to as "flavour tagging", and is a technique employed in a variety of experiments [1]. In the case of ϕ- and B-factories, where the neutral meson states produced (K^0–$\overline{K}^0$ and B^0–$\overline{B}^0$, respectively) constitute correlated Einstein-Podolsky-Rosen states (EPR) [3, 4, 5], the knowledge that one of the two mesons decays at a given time through a flavour specific channel ("tagging") allows one to unambiguously infer the flavour of the accompanying meson state at the same time. Thus, for example, the detection of a flavour specific B^0 (or $\overline{B}^0$) decay on one side of the detector implies a pure $\bar{B}^0$ (or B^0) state on the other side (we will always refer to the fra me associated with the resonance, the center-of-mass frame). In this article we will argue that the basic underlying (and usually unquestioned) assumptions, leading to the above conclusion, may be invalidated if the CPT operator cannot be *intrinsically* defined. These latter circumstances may occur, for example, in the context of an extended class of quantum gravity models, where the structure of quantum space time at Planckian scales (10^{-35} m) may actually be fuzzy, characterised by a "foamy" nature (*space time foam*) [6]. In addition, we will propose a set of basic observables, whose measurement would effectively amount to a direct testing of the validity of the hypothesis associated with the tagging.

In what follows we will go over the assumptions built into the flavour tagging with EPR states. In the conventional formulations of *entangled* meson states [3, 4, 5] one imposes the requirement of *Bose statistics* for the state $K^0\overline{K}^0$ or $B^0\overline{B}^0$. This, in turn, implies that the physical neutral meson-antimeson state must be *symmetric* under the combined operation C$\mathcal{P}$, where C is the charge conjugation and $\mathcal{P}$ the operator that permutes the spatial coordinates. Specifically, assuming *conservation* of angular momentum, and a proper existence of the *antiparticle state* (denoted by a "bar"), one observes that, for $K^0\overline{K}^0$ states which are C-selfconjugates with C= $(-1)^\ell$ (with ℓ the angular momentum quantum number), the system has to be an eigenstate of $\mathcal{P}$ with eigenvalue $(-1)^\ell$. Hence, for $\ell = 1$, we have that C= $-$, implying $\mathcal{P} = -$. As a consequence of Bose statistics this ensures that for $\ell = 1$ the state of two identical bosons is forbidden [3]. What is more, the probability of obtaining identical decay channels at equal times exactly vanishes, independently of CP, T and/or CPT violation in the effective hamiltonian.

As a result, the initial entangled state $B^0\overline{B}^0$ produced in a B factory can be written as:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} \left(|B^0(\vec{k}), \overline{B}^0(-\vec{k})\rangle - |\overline{B}^0(\vec{k}), B^0(-\vec{k})\rangle \right),$$ (1)

where B-meson momenta are $\pm\vec{k}$ and $\vec{k}\cdot\vec{p}_{e-} > 0$, with $\vec{p}_{e-}$ the momentum of the colliding e^-. This specific form of the state vector is intimately connected with the procedure of tagging. In particular, the antisymmetric nature of the state under permutation forbids the presence of $|B^0 B^0\rangle$ and $|\overline{B}^0\overline{B}^0\rangle$ terms. It is elementary to verify that, under normal Hamiltonian evolution of the system, this latter property persists: at any given time the state remains antisymmetric, given by

$$|\psi(t)\rangle = \frac{e^{-iMt-\frac{\Gamma}{2}t}}{\sqrt{2}} \left\{ |B^0\overline{B}^0\rangle - |\overline{B}^0 B^0\rangle \right\}.$$ (2)

Evidently, detection of a given flavour at any time t implies the presence of the opposite flavour at the same time and in opposite sides of the detector.

However, as has been pointed out for the first time in [7], the assumptions leading to Eq.(1) may not be valid if CPT symmetry is violated, not in the usually considered sense of the CPT operator not commuting with the hamiltonian of the system at hand [8], but rather in a way which most likely occurs in quantum gravity. Namely, a decoherent quantum evolution takes place in the "medium" of a space time foam [6], in which case pure states evolve into mixed ones, a scattering S-matrix cannot be properly defined, and hence, according to the theorem of Ref. [9], the CPT operator is not well defined, thereby leading to a strong form of CPT violation. In such a case [7] $\overline{B}^0$ cannot be considered as identical to B^0, and thus the requirement of $C\mathcal{P} = +$, imposed by Bose statistics, is relaxed. As a result, the initial entangled state (1) can be parametrised in general as:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2\left(1+|\omega|^2\right)}} \left\{ |B^0\overline{B}^0\rangle - |\overline{B}^0 B^0\rangle + \omega \left[|B^0\overline{B}^0\rangle + |\overline{B}^0 B^0\rangle \right] \right\},$$ (3)

where $\omega = |\omega|e^{i\Omega}$ is a *complex* CPT-violating parameter [7], associated with the non-identical particle nature of the neutral meson and antimeson states. We emphasize that the modification in Eq.(3) is due to the loss of indistinguishability of B^0 and $\overline{B}^0$ and not due to violation of symmetries in the production process. Evidently, the probabilities for the two states connected by a permutation are different due to the presence of ω.

This modification of the initial state vector has far-reaching consequences for the concept of particle tagging. In what follows we will study the time evolution of (3), in order to (i)

establish the appearance of terms of the (previously forbidden) type $|B^0 B^0\rangle$ and $|\overline{B}^0 \overline{B}^0\rangle$, and (ii) introduce a set of observables, which could actually serve as a direct way for checking experimentally the robustness of the correlation between the two states assumed during the tagging.

The eigenstates of the effective hamiltonian with well defined time evolution are

$$|B_1\rangle = \frac{1}{\sqrt{2\left(1+|\epsilon_1|^2\right)}} \left((1+\epsilon_1)|B^0\rangle + (1-\epsilon_1)|\overline{B}^0\rangle\right),$$

$$|B_2\rangle = \frac{1}{\sqrt{2\left(1+|\epsilon_2|^2\right)}} \left((1+\epsilon_2)|B^0\rangle - (1-\epsilon_2)|\overline{B}^0\rangle\right). \tag{4}$$

The eigenvalues of the effective hamiltonian corresponding to $|B_1\rangle$ and $|B_2\rangle$ are, respectively, $\mu_1 = M_1 + i\Gamma_1/2$ and $\mu_2 = M_2 + i\Gamma_2/2$, and we define the quantities $M = (M_1 + M_2)/2$, $\Delta M = M_1 - M_2$, $\Gamma = (\Gamma_1 + \Gamma_2)/2$, and $\Delta\Gamma = \Gamma_1 - \Gamma_2$.

Thus, written in terms of the states $|B_1\rangle$ and $|B_2\rangle$, the initial state $|\psi(0)\rangle$ in Eq.(3) assumes the form

$$|\psi(0)\rangle = \frac{1}{\sqrt{2\left(1+|\omega|^2\right)}} \left\{ C_{12}|B_1 B_2\rangle + C_{21}|B_2 B_1\rangle + C_{11}|B_1 B_1\rangle + C_{22}|B_2 B_2\rangle \right\}, \tag{5}$$

where

$$C_{12} = \frac{\sqrt{(1+|\epsilon_1|^2)(1+|\epsilon_2|^2)}}{\epsilon_1 \epsilon_2 - 1} \left(1 - \omega \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 \epsilon_2 - 1}\right),$$

$$C_{21} = -\frac{\sqrt{(1+|\epsilon_1|^2)(1+|\epsilon_2|^2)}}{\epsilon_1 \epsilon_2 - 1} \left(1 + \omega \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 \epsilon_2 - 1}\right),$$

$$C_{11} = \omega \frac{(1-\epsilon_2^2)(1+|\epsilon_1|^2)}{(1-\epsilon_1\epsilon_2)^2},$$

$$C_{22} = -\omega \frac{(1-\epsilon_1^2)(1+|\epsilon_2|^2)}{(1-\epsilon_1\epsilon_2)^2}. \tag{6}$$

We note the presence of $|B_1 B_1\rangle$ and $|B_2 B_2\rangle$, which is a characteristic feature when $\omega \neq 0$. Furthermore, $C_{12} \neq -C_{21}$.

With the quantum mechanical effective Hamiltonian time evolution, the states at a later time t are given by

$$|B_1(0)\rangle \mapsto e^{-iMt-\frac{\Gamma}{2}t} e^{-i\frac{\Delta M}{2}t - \frac{\Delta\Gamma}{4}t} |B_1(0)\rangle, \quad |B_2(0)\rangle \mapsto e^{-iMt-\frac{\Gamma}{2}t} e^{+i\frac{\Delta M}{2}t + \frac{\Delta\Gamma}{4}t} |B_2(0)\rangle, \tag{7}$$

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implying that the various terms in Eq.(5) will have in general a different time evolution. Returning to the flavour-state basis, since we are interested in flavour specific decay channels, we have

$$|\psi(t)\rangle = \frac{e^{-iMt-\frac{\Gamma}{2}t}}{\sqrt{2\left(1+|\omega|^2\right)}} \left\{ C_{0\bar{0}}(t)|B^0\overline{B}^0\rangle + C_{\bar{0}0}(t)|\overline{B}^0 B^0\rangle + C_{00}(t)|B^0 B^0\rangle + C_{\bar{0}\bar{0}}(t)|\overline{B}^0\overline{B}^0\rangle \right\},$$

(8)

where

$$C_{0\bar{0}}(t) = 1 + \omega f(t),$$

$$C_{\bar{0}0}(t) = -1 + \omega f(t),$$

$$C_{00}(t) = \frac{\omega}{\left(1-\epsilon^2+\frac{\delta^2}{4}\right)^2} \left((1+\epsilon)^2 - \frac{\delta^2}{4}\right) \left(f_1(t) + f_2(t)\right),$$

$$C_{\bar{0}\bar{0}}(t) = \frac{\omega}{\left(1-\epsilon^2+\frac{\delta^2}{4}\right)^2} \left((1-\epsilon)^2 - \frac{\delta^2}{4}\right) \left(f_1(t) - f_2(t)\right),$$

(9)

with

$$f(t) = \frac{1}{\left(1-\epsilon^2+\frac{\delta^2}{4}\right)^2} \left[\delta^2 + \frac{1}{2}\left((1+\epsilon)^2 - \frac{\delta^2}{4}\right)\left((1-\epsilon)^2 - \frac{\delta^2}{4}\right)\left(e^{\alpha t} + e^{-\alpha t}\right)\right],$$

$$f_1(t) = -\frac{1}{2}\left(1-\epsilon^2+\frac{\delta^2}{4}\right)\left(e^{\alpha t} - e^{-\alpha t}\right),$$

$$f_2(t) = -\delta + \frac{\delta}{2}\left(e^{\alpha t} + e^{-\alpha t}\right),$$

(10)

and we have defined $\epsilon = (\epsilon_1 + \epsilon_2)/2$, $\delta = \epsilon_1 - \epsilon_2$, and $\alpha \equiv i\Delta M/2 + \Delta\Gamma/4$. We emphasize that the above expressions are exact; no expansion with respect to any of the parameters has taken place. Phase redefinitions of the single B-meson states such as $B^0 \mapsto e^{i\gamma}B^0$, $\overline{B}^0 \mapsto e^{i\bar{\gamma}}\overline{B}^0$ are easily handled through the transformation of $\{\epsilon,\delta\}$-dependent expressions. The most useful properties under the above mentioned rephasings are

$$\frac{\delta}{1-\epsilon^2+\frac{\delta^2}{4}} \mapsto \frac{\delta}{1-\epsilon^2+\frac{\delta^2}{4}} \quad ; \quad \frac{(1\pm\epsilon)^2 - \frac{\delta^2}{4}}{1-\epsilon^2+\frac{\delta^2}{4}} \mapsto \frac{(1\pm\epsilon)^2 - \frac{\delta^2}{4}}{1-\epsilon^2+\frac{\delta^2}{4}} e^{\pm i(\gamma-\bar{\gamma})}.$$

They lead to explicitly rephasing invariant $C_{0\bar{0}}(t)$ and $C_{\bar{0}0}(t)$ coefficients, whereas $C_{00}(t) \mapsto e^{i(\gamma-\bar{\gamma})}C_{00}(t)$ and $C_{\bar{0}\bar{0}}(t) \mapsto e^{i(\bar{\gamma}-\gamma)}C_{\bar{0}\bar{0}}(t)$ are individually rephasing-variant, but their dependence on the phase is such that the considered physical observables are rephasing invariant, as they should. As a check we note that setting $t = 0$ in the above expressions for C_{ab} we recover the state of Eq.(3).

Evidently, a non-vanishing ω allows both symmetric and antisymmetric terms under $B^0 \leftrightarrows \overline{B}^0$. Thus, contrary to the standard $\omega = 0$ case where the antisymmetric nature of the state forbids the presence of $|B^0 B^0\rangle$ and $|\overline{B}^0 \overline{B}^0\rangle$ terms, both $|B^0 B^0\rangle$ and $|\overline{B}^0 \overline{B}^0\rangle$ terms appear at $t \neq 0$. This result has an important consequence on the concept of flavour tagging: in the presence of the ω effect, the detection of a flavour specific B^0 (or $\overline{B}^0$) decay on one side does not necessarily imply a pure $\bar{B}^0$ (or B^0) state on the other side. Clearly, there is a minute "contamination", due to the presence on one side of the same meson that has been actually tagged at the opposite side.

Having concluded the demise of the concept of tagging in the presence of ω, it would be interesting to invent a set of observables which would actually measure the deviation, if any, from the basic tagging assumption. We will focus on observables involving *simultaneous* B^0 or $\overline{B}^0$ flavour specific decays. This eliminates the standard terms $B^0(t) B^0(t + \Delta t)$ and $\overline{B}^0(t) \overline{B}^0(t+\Delta t)$ as they vanish for $\Delta t = 0$. In what follows we will restrict our attention to the most characteristic case of flavour specific channels, namely semileptonic decays. The main reason for this choice is the fact that the flavour specificity of such decays relies on a minimum number of assumptions, in particular solely on the equality $\Delta B = \Delta Q$, and is completely independent of whether or not the CP and CPT symmetries are exact [15]. We emphasize that other flavour specific channels may not share this property when there is CP or CPT violation in the decay. Notice also that any effects stemming from the possibly decoherent (i.e. non quantum-mechanical) evolution of the initial state can be unambiguously separated from the ω effect through the difference in the symmetry properties of their contributions to the density matrix [7].

Our basic observables are equal time intensities of flavour specific decays of B mesons. We consider the four flavour specific channels $|X_{00}\rangle$, $|X_{\bar{0}\bar{0}}\rangle$, $|X_{0\bar{0}}\rangle$ and $|X_{\bar{0}0}\rangle$, characteristic to the B-meson combinations $|B^0 B^0\rangle$, $|\overline{B}^0 \overline{B}^0\rangle$, $|B^0 \overline{B}^0\rangle$, and $|\overline{B}^0 B^0\rangle$, respectively. Since, $\left| \langle X_{ab} | B^c B^d \rangle \right| \sim \delta_a^c \delta_b^d$, with $ab, cd = 00, \bar{0}\bar{0}, 0\bar{0}, \bar{0}0$ (in this compact notation, $|\overline{B}^0\rangle \equiv |B^{\bar{0}}\rangle$, etc), it is evident that sandwiching the state of Eq.(8) with one of the aforementioned flavour specific channels projects out the corresponding co-factor C_{ab}. Defining the four intensities $I_{ab}(t) = |\langle X_{ab} | \psi(t) \rangle|^2$ we find that

$$I_{ab}(t) = |\langle Y_a | B^a \rangle|^2 \, |\langle Z_b | B^b \rangle|^2 \, \frac{e^{-\Gamma t}}{2(1 + |\omega|^2)} \, |C_{ab}(t)|^2 \,, \tag{11}$$

where the state $|X_{ab}\rangle$ has been decomposed into the two single-meson flavour-specific decay

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states, Y_a and Z_b, i.e. $|X_{ab}\rangle = |Y_a, Z_b\rangle$. These equal-time intensities can be easily time-integrated:

$$\mathcal{I}_{ab} = \int_0^\infty dt \; I_{ab}(t). \tag{12}$$

As seen in Eq. (9), the parameters involved in the time evolution, $\epsilon, \delta, \Delta M, \Delta \Gamma$ only appear in terms which are explicitly proportional to ω. For the $B^0 - \overline{B}^0$ system, the terms proportional to $\omega \delta$ and $\omega \Delta \Gamma$ can be considered as higher order.

In terms of intensities, $\omega \neq 0$ allows

$$I_{00}(t) \neq 0 \quad ; \quad I_{\bar{0}\bar{0}}(t) \neq 0 \; .$$

It is through these otherwise (for $\omega = 0$) forbidden intensities that we can explore the presence of $\omega \neq 0$. As we can see in Eq.(9) and Eq.(11), what one hopes to observe is an $|\omega|^2$ vs. 0 effect. This would be an *unambiguous* manifestation of our effect, independently of any other source of symmetry violation.

In the hypothetical situation of non-vanishing values for $I_{00}(t)$ and $I_{\bar{0}\bar{0}}(t)$ one could consider a CP-type asymmetry of the form

$$A_{CP}(t) = \frac{I_{00}(t) - I_{\bar{0}\bar{0}}(t)}{I_{00}(t) + I_{\bar{0}\bar{0}}(t)} \quad ; \quad \mathcal{A}_{CP} = \frac{\mathcal{I}_{00} - \mathcal{I}_{\bar{0}\bar{0}}}{\mathcal{I}_{00} + \mathcal{I}_{\bar{0}\bar{0}}} \quad . \tag{13}$$

The asymmetries $A_{CP}(t)$ and $\mathcal{A}_{CP}$ express the difference between the decay rates of $B^0 \rightarrow X_0$ and $\overline{B}^0 \rightarrow X_{\bar{0}}$, where, as before, X_0 is a specific flavour channel and $X_{\bar{0}}$ its C-conjugate state (in our notation $\overline{X}_0 \equiv X_{\bar{0}}$). In order to isolate the physics associated with C_{00} and $C_{\bar{0}\bar{0}}$ through an observable such as $A_{CP}(t)$, one must eliminate its dependence on the decay amplitudes $|\langle Y_a | B^a \rangle|^2 |\langle Z_b | B^b \rangle|^2$ entering through Eq.(11). If the physics governing the decay is CPT-invariant (as in the Standard Model), the use of inclusive channels guarantees the cancellation of the decay amplitudes in Eq.(13). If we consider exclusive channels instead, CP violation in the decays prevents in general the aforementioned cancellation from taking place, thus restricting the usefulness of $A_{CP}(t)$. In addition to these standard considerations, quantum gravity itself may affect the CPT invariance in the decays; nevertheless, such contributions will be subleading, and we will neglect them in what follows.

Interestingly enough, $A_{CP}(t)$ and $\mathcal{A}_{CP}$ are independent of the value of ω, since the latter clearly cancels out when forming the corresponding ratios, to leading order, when quantum gravity induced CPT violating effects in the decays are ignored. For $\delta = 0$ and $\Delta \Gamma$ small,

such that terms of order $\omega\Delta\Gamma$ can be safely neglected, Eq.(13) simplifies to

$$A_{CP}(t) = \mathcal{A}_{CP} = \frac{|1+\epsilon|^4 - |1-\epsilon|^4}{|1+\epsilon|^4 + |1-\epsilon|^4} = \frac{4\,(1+|\epsilon|^2)\,\Re e\,\epsilon}{(1+|\epsilon|^2)^2 + (2\,\Re e\,\epsilon)^2}\;. \tag{14}$$

In terms of the standard mixing parameters p and q,

$$\frac{|1+\epsilon|^4 - |1-\epsilon|^4}{|1+\epsilon|^4 + |1-\epsilon|^4} = \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4} = \frac{2\Delta_B}{1 + \Delta_B^2}\;,$$

where

$$\Delta_B = \frac{2\,\Im m(M_{12}^*\Gamma_{12})}{(\Delta M)^2 + |\Gamma_{12}|^2}.$$

According to the present measurements of the semileptonic rate asymmetry [11], $A_{CP}(t) = \mathcal{A}_{CP} = -0.007 \pm 0.013$.

The algebraic cancellation of all the ω dependence in Eq.(13) can be physically understood by realizing that $\omega \neq 0$ allows the equal time presence of $|B^0 B^0\rangle$ and $|\overline{B}^0\overline{B}^0\rangle$ terms, and it has nothing to do with B^0–$\overline{B}^0$ mixing or $B^0, \overline{B}^0$ decays. As mentioned previously, possible quantum gravity effects in the decays contribute to higher order (at least linear in ω-like parameters) terms in Eq.(14). The CP asymmetries in Eq.(13) are thus *conventional* CP asymmetries between states which are both CPT-forbidden; this cancellation is an explicit proof of both effects. This provides an additional way of testing the self-consistency of the entire procedure: Once non-vanishing $I_{00}(t)$ and $I_{\bar{0}\bar{0}}(t)$ have been established one should extract the experimental value of $\mathcal{A}_{CP}$, which should coincide with the theoretical expression of Eq.(13); for the calculation of the latter one needs as input only the standard value for the parameter ϵ, with no reference to the actual value of ω.

To isolate linear effects in ω, we pay attention to the channels "$0\bar{0}$" and "$\bar{0}0$" and consider the following CPT-violating, *exchange asymmetries*:

$$A(t) = \frac{I_{0\bar{0}}(t) - I_{\bar{0}0}(t)}{I_{0\bar{0}}(t) + I_{\bar{0}0}(t)} \quad ; \quad \mathcal{A} = \frac{\mathcal{I}_{0\bar{0}} - \mathcal{I}_{\bar{0}0}}{\mathcal{I}_{0\bar{0}} + \mathcal{I}_{\bar{0}0}}\;. \tag{15}$$

As in $A_{CP}(t)$ and $\mathcal{A}_{CP}$, we are interested in eliminating, in Eq.(15), the effects related to the decays: this is again accomplished through CPT-invariant inclusive semileptonic decays or CP-conserving flavour specific hadronic channels. As we shall explain below, $A(t)$ and $\mathcal{A}$ measure the difference between the amplitudes corresponding to the permuted states $|B^0\overline{B}^0\rangle$ and $|\overline{B}^0 B^0\rangle$.

To this end, we find it instructive to clarify first some crucial concepts with the help of figure 1, which depicts inclusive semileptonic B decays, for definiteness. For our purposes,

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the situation is identical to flavour-specific hadronic channels. When $\omega = 0$, the states $|B^0\overline{B}^0\rangle$ and $|\overline{B}^0 B^0\rangle$ are related through charge conjugation C and through the permutation $B^0 \leftrightarrows \overline{B}^0$; as a consequence of Bose symmetry, *no observable* can distinguish between those states. Notice that this fact does not rely on the definition of two-particle states. Indeed, recall that $|B^0\overline{B}^0\rangle$ stands for $|B^0(\vec{k})\overline{B}^0(-\vec{k})\rangle$, where, as pointed out after Eq.(1), $\vec{k}$ is such that $\vec{k}\cdot\vec{p}_{e^-} > 0$ (this implies $0 \le \theta < \frac{\pi}{2}$ for the situation depicted in fig. 1). The schematic events shown in the figure correspond unambiguously to the two-particle state that is actually projected out:

$$1(a) \rightarrow |B^0\overline{B}^0\rangle \quad ; \quad 1(b) \rightarrow |\overline{B}^0 B^0\rangle \quad ; \quad 1(c) \rightarrow |\overline{B}^0 B^0\rangle \quad ; \quad 1(d) \rightarrow |B^0\overline{B}^0\rangle .$$

When $\omega = 0$, the identity $I_{0\bar{0}}(t) = I_{\bar{0}0}(t)$ is independent of our $\vec{k}$-dependent two particle convention. The situation changes drastically when $\omega \neq 0$. First of all, notice that the pairs (1(a) and 1(d)) and (1(b) and 1(c)) in fig. 1 are related through charge conjugation C, while the pairs (1(a) and 1(c)) and (1(b) and 1(d)) are related through permutations $B^0 \leftrightarrows \overline{B}^0$.
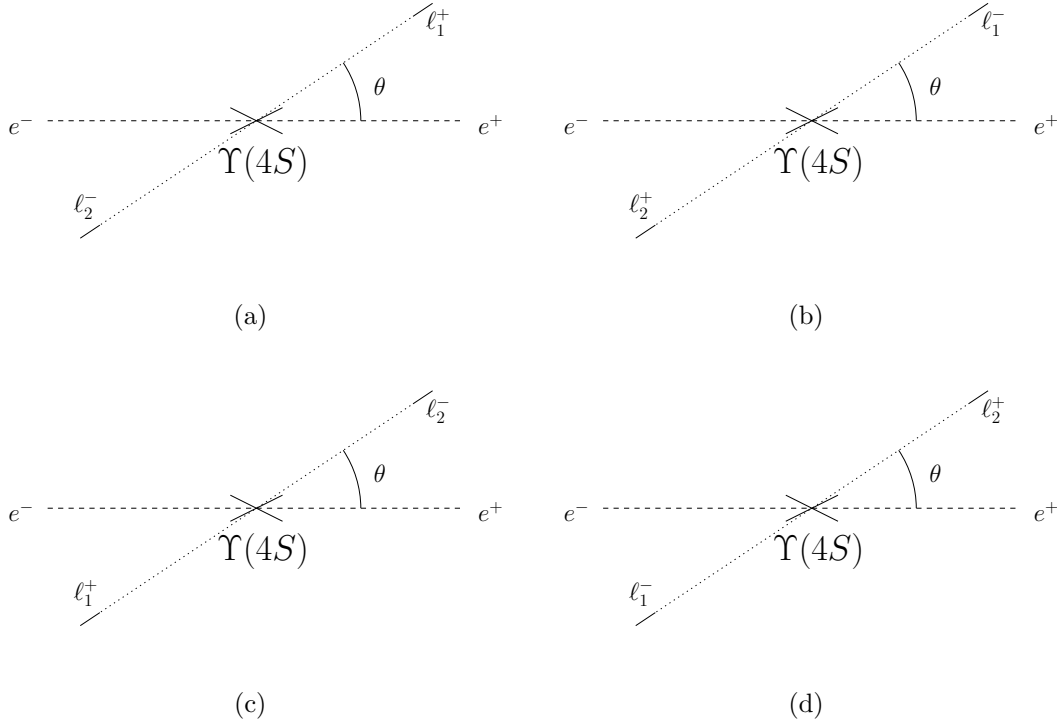

FIG. 1: Schematic events in inclusive semileptonic B decays. The leptons are the only final particles shown, for brevity.

As the permutation $B^0 \leftrightarrows \overline{B}^0$ is no longer a symmetry, any sensible definition of two-particle states should not include contributions related through the permutation $B^0 \leftrightarrows \overline{B}^0$ in the same intensity $I_{0\bar{0}}(t)$ or $I_{\bar{0}0}(t)$. Note also that there is an invariance of these intensities under rotations around the colliding $e^- e^+$ direction. Our $\vec{k}$-dependent definition is the simplest one that guarantees these properties. Indeed, events of the type 1(a) and 1(d) contribute to $I_{0\bar{0}}(t)$, while events like 1(b) and 1(c) contribute to $I_{\bar{0}0}(t)$. Under charge conjugation, $I_{0\bar{0}}(t) \to I_{0\bar{0}}(t)$ and $I_{\bar{0}0}(t) \to I_{\bar{0}0}(t)$, whilst under $B^0 \leftrightarrows \overline{B}^0$, $I_{0\bar{0}}(t) \to I_{\bar{0}0}(t)$ and $I_{\bar{0}0}(t) \to I_{0\bar{0}}(t)$. From the above discussion, then, it becomes clear that $A(t)$ and $\mathcal{A}$ measure the asymmetry originated by the permutation $B^0 \leftrightarrows \overline{B}^0$.

Using the expressions for C_{ab} given in Eq.(9), it is straightforward to establish that $A(t)$ depends, to leading order, linearly on ω, due to the interference between the ω-dependent and the standard, ω-independent ("1"), parts of $C_{0\bar{0}}(t)$ and $C_{\bar{0}0}(t)$:

$$A(t) = \frac{2\Re e\left(\omega f(t)\right)}{1 + |\omega f(t)|^2} \quad . \tag{16}$$

where $f(t)$ is defined in (10).

For $\delta = 0$ and $\Delta\Gamma \to 0$, Eq.(16) simplifies to

$$A(t) = \frac{2\,\Re e(\omega)\,\cos\left(\Delta M\,t/2\right)}{1 + |\omega|^2 \cos^2\left(\Delta M\,t/2\right)} \quad ; \quad \mathcal{A} = \frac{2\Gamma^2}{\Gamma^2 + \left(\frac{\Delta M}{2}\right)^2} \frac{\Re e(\omega)}{1 + \mathcal{F}(|\omega|^2)} \quad , \tag{17}$$

where $\mathcal{F}(|\omega|^2) = \frac{1}{2} |\omega|^2 \frac{2\Gamma^2 + (\Delta M)^2}{\Gamma^2 + (\Delta M)^2}$.

This concludes our analysis on the ω-effect-induced demise of flavour tagging in B-meson factories. We stress once more that the above-described effects are specific to a particular kind of CPT violation invoking decoherence, which affects the *identity* of the (initial) neutral meson states [7], and is in principle unrelated to the dynamics of their evolution. This is clearly distinguishable from other types of CPT violation existing in the literature, e.g. those pertaining to the non-commutativity of the CPT operator with the matter hamiltonian [12], or those related to non-local field theory models [13], or even those associated with a decoherent temporal *evolution* of matter in quantum gravity media [14]. It is hoped that studies in B-factories such as the one suggested above will improve the bounds on such effects significantly in the foreseeable future. Together with other neutral meson factories, such as ϕ-factories [7], this system may then provide essential probes for novel physics, associated with effects of quantum gravity on entangled states. This should be viewed as complementary to other quantum gravity studies.

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[1] see for instance: R. J. Wilson, "Neutral B meson flavor tagging,", *Prepared for 5th International Linear Collider Workshop (LCWS 2000), Fermilab, Batavia, Illinois, 24-28 Oct 2000*, and references therein; see also: H. Kakuno *et al.*, arXiv:hep-ex/0403022.

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